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Mode Cutoff Frequencies in Screened Dielectric Rod Waveguide*

GLOSSARY OF SYMBOLS

ρ, ϕZ = cylindrical coordinates
a = radius of dielectric rod
b = radius of metal screen
$\omega = 2\pi \times$ frequency
μ_r = relative permeability of medium
ϵ_r = relative permittivity of medium
μ_0 = permeability of free space
ϵ_0 = permittivity of free space
$\mu' = \mu_0\ \mu_r$
$\epsilon' = \epsilon_0\epsilon_r$
E, H = electromagnetic field vectors
β = complex component of propagation constant
h = wave number $= (\gamma^2 + \omega^2\mu'\epsilon')^{1/2}$
λg = wavelength in waveguide
λ = free space wavelength
f = frequency
$J_n(x)$ = Bessel function of first-kind order n
$Y_n(x)$ = Bessel function of second-kind order n
$H_n^{(1)}(x)$ = Bessel function of third-kind order n
$Z_n'(x)$ = first derivative of Bessel function with respect to x

The geometry of the screened dielectric rod waveguide is shown in Fig. 1. The theory for this waveguide has been solved by Beam and Wachowski[1] for the case of $\mu_1 = \mu_2 = 1$. For the general case when $\mu_1 \neq \mu_2 \neq 1$ the characteristic equation becomes

$$
\begin{aligned}
&n^2K^2\beta^2(q_2q_4 - q_3q_6)(q_3 - q_4)\\
&+ r^4\omega^2\mu_0\epsilon_0\left[\{\epsilon_1q_7(q_4 - q_3) + \epsilon_2(q_1q_3 - q_4q_5)\}\right.\\
&\left.\cdot\{\mu_1q_7(q_2q_4 - q_3q_6) - \mu_2(q_2q_4q_5 - q_1q_3q_6)\}\right] = 0
\end{aligned}
\tag{1}
$$

where

$$
k = \frac{1}{(h_1b)^2} - \frac{1}{(h_2b)^2}, \qquad r = \frac{a}{b}
$$

$$
q_1 = \frac{J_n'(h_2a)}{h_2aJ_n(h_2a)}, \qquad q_2 = \frac{J_n'(h_2b)}{h_2bJ_n(h_2b)}
$$

$$
q_3 = \frac{J_n(h_2a)}{Y_n(h_2a)}, \qquad q_4 = \frac{J_n(h_2b)}{Y_n(h_2b)}
$$

$$
q_5 = \frac{Y_n'(h_2a)}{h_2a\,Y_n(h_2a)}, \qquad q_6 = \frac{Y_n'(h_2b)}{h_2b\,Y_n(h_2b)}
$$

$$
q_7 = \frac{J_n'(h_1a)}{h_1aJ_n(h_1a)}.
$$

At cutoff $\beta = 0$ and two equations result:

$$
\epsilon_1q_7(q_4 - q_3) - \epsilon_2(q_4q_5 - q_1q_3) = 0 \tag{2}
$$

$$
\mu_1q_7(q_2q_4 - q_3q_6) - \mu_2(q_2q_4q_5 - q_1q_3q_6) = 0. \tag{3}
$$

At cutoff $h = 2\pi\sqrt{\mu_r\epsilon_r}/\lambda$.

These two equations can be solved graphically by inserting the required values of ϵ, μ, and λ. Eq. (2) gives the cutoff condition for E-type modes and (3) gives the condition for H-type modes.

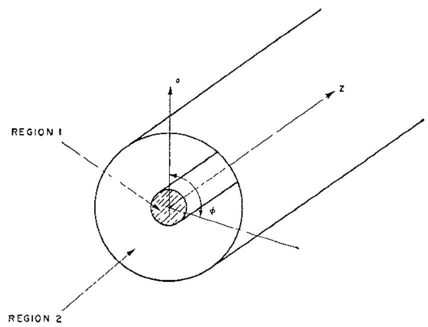

Fig. 1—Waveguide geometry.

TABLE I

Mode	$\frac{b}{c}$	$\frac{b^{(1)}}{c}$
HE_{11}	0.269	0.27
E_{01}	0.305	0.30
HE_{21}	0.477	—
EH_{11}	0.556	—
H_{01}	0.568	0.56

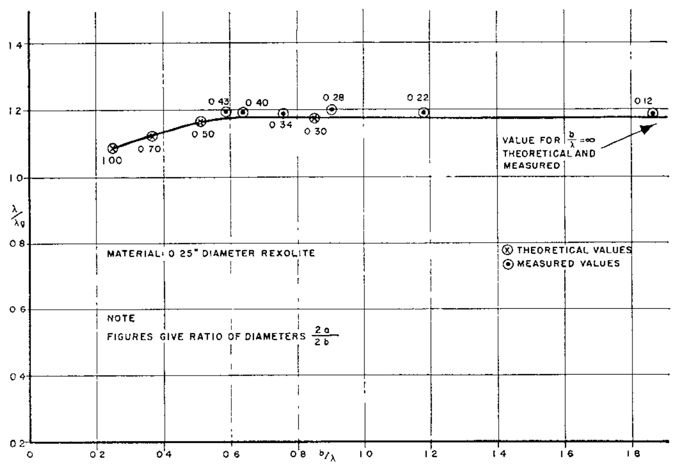

Fig. 2—Theoretical and measured guide wavelengths.

These equations have been solved for the first five modes in a screened dielectric rod waveguide for $\epsilon_1 = 2.5$ and $r = 0.3$. Values of b/λ_c are given in Table I where they are compared with the graphical values of Beam and Wachowski. The mode nomenclature gives the order of the Bessel function and the order of the root of (2) or (3). The mode has been termed HE if solved by (3) for $n \neq 0$ and EH if solved by (2) for $n \neq 0$.

It can be seen that the dielectric rod reduces the cutoff frequency of the EH_{11} mode so that the E_{11} limit mode and the H_{01} mode no longer have coincident cutoff frequencies. It is also interesting to note that the effect of the dielectric rod of $r = 0.3$ on the H_{21} mode is very small. (For the H_{21} mode $b/\lambda_c = 0.486$.) This is consistent with the mode pattern of the H_{21} mode.

Some measurements of the HE_{11} mode wavelength in a screened dielectric rod waveguide have been made for 0.25-in rexolite rod ($\epsilon_r = 2.5$) at 24,000 Mc. Fig. 2 shows that above a particular value of b/λ

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[1] R. E. Beam and H. M. Wachowski, "Shielded Dielectric Rod Waveguide," Microwave Lab., Northwestern University, Chicago, Ill., pp. 270–309; 1950 (Final rept. on investigation of multimode propagation in waveguides and microwave optics.)

the guide wavelength remains constant as the screen diameter is increased. Thus the effect of the screen on the HE_{11} mode becomes small and the wave propogates in a quasi-dielectric rod mode. However, (2) and (3) show that higher-order modes are also produced by the screen and a pure quasi-dielectric rod HE_{11} mode is not propagated. The theoretical results in Fig. 2 were taken from Beam and Wachowski.[1]

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On the Measurement of Detector Impedance*

A detector or receiver may be used in conjunction with a directional coupler and a calibrated variable short to measure its own impedance. The observed VSWR as the short is adjusted is the VSWR of the detector if the coupling is light. Various minor corrections are described.

The purpose of this paper is to describe a method for the measurement of the impedance of a detector of known response law without the use of an auxiliary detector. The method is especially suitable for use at the lowest power level within its range of operation. The detector could be a crystal, a bolometer or a receiver. The method is related to the resonance curve (Chipman method) impedance measuring technique. It is also related to the usual method of measuring impedance of a receiver in which the source and detector of a standard setup are exchanged.[1] The unusual feature is that the detector is used to measure its own impedance.[2]

It is assumed, of course, that the detector law is known and that its impedance does not change for the moderate change of received power levels encountered during the VSWR measurements. The method utilizes a directional coupler, a matched load, and a moving short with a position indicator, in addition to the signal source and the detector. A slotted line is not required. The method can be applied to standard microwave circuits but has been used primarily in circuits utilizing plane waves in the submillimeter wavelength range.[3]

Let a signal source and a matched termination be connected to the main arms of a directional coupler, the detector to be tested to the forward-coupled side arm and a calibrated shorting (tuning) plunger to the reverse arm, as sketched in Fig. 1. Let the reflection coefficient of the mount be Γ and the corresponding transmission coefficient to the detector element itself be T. The voltage transmission coefficient of the coupler is t and its voltage coupling coefficient is r. The shorting plunger is assumed perfect and has a reflection coefficient $e^{j\beta}$.

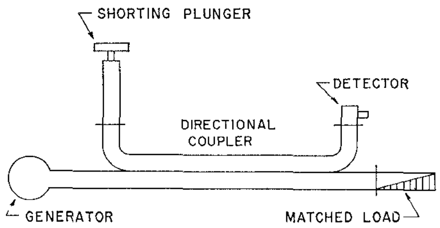

Fig. 1—Circuit for measuring detector impedance.

An incident wave of unit amplitude, upon reaching the coupler, will be partially transmitted to the detector. It will deliver a voltage rT to the detector itself. The reflection from the detector, of magnitude $r\Gamma$, will pass twice through the coupler on its way to the shorting plunger and back. It will contribute an additional voltage $r\Gamma t^2 e^{j\beta} T$ at the detector. Multiple reflections will exist; the total signal delivered to the detector will be

$$E = \frac{rT}{1 - t^2 e^{j\beta}}.$$

Maxima and minima will be observed as the shorting plunger is adjusted to make the denominator real. Their ratio is:

$$\rho = \frac{1 + t^2 |\Gamma|}{1 + t^2 |\Gamma|},$$

from which we obtain the reflection coefficient of the detector mount:

$$|\Gamma| = \frac{\rho - 1}{t^2(\rho + 1)}.$$

For light coupling, $t \approx 1$ and ρ will be approximately the voltage standing-wave ratio of the mount.

The measurement, of course, can be no more accurate than the equipment used. In the equations above, it has been assumed that the directivity of the coupler is infinite and that the matched load is perfect. Imperfections will have an effect similar to that of the residual VSWR of a slotted line.

It will be noted that no auxiliary detector is used in this measurement. It is required only that the signal source be moderately well-matched and that the transmission coefficient t of the coupler be known. The loss in the shorting plunger may also be lumped in with t if it has been measured. Note that the losses in the mount have not been measured. They affect the value of T which is eliminated in the ratio of maximum to minimum signals. The mount efficiency must be measured by an appropriate series of measurements of mount impedance.

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[1] E. L. Ginzton, "Microwave Measurements," McGraw-Hill Book Co., Inc., New York, N. Y., pp. 287 ff., 307; 1957.

[2] The authors are indebted to R. W. Beatty for calling to their attention a related technique in which an iris mismatch at the generator and a squeezable waveguide section replace the directional coupler and the sliding short used in this paper. The technique was described by L. S. Liberman, "A method of measuring VSWR of video detectors," *Radiotekhnika i elektronika*, vols. 2, no. 7, pp. 941–942, 1957; *Radio Eng'g. and Electronics*, vol. 2, p. 180, 1957. (English translation.)

[3] R. H. Miller, P. A. Szente, and K. B. Mallory, "A Measurement of Bolometer Mount Efficiency at Millimeter Wavelengths," presented at the Millimeter and Submillimeter Conf., Orlando, Fla.; January, 1963.

The Synthesis of N-Port Circulators*

Professor P. Penfield has kindly brought to our notice his paper on lossless three-ports,[1] which independently covers some of the work described by us.[2] He has pointed out that for circulator synthesis from a three-port junction only one of the three conditions of equation (16) or (17) need be specified. This can be seen from the first two equations of (9) of our paper. Otherwise our classifications of three-port networks are the same.

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* Received April 5, 1963.

[1] P. Penfield, Jr., "A classification of lossless three-ports," IRE TRANS. ON CIRCUIT THEORY, vol. CT-9, pp. 215–223; September, 1962.

[2] B. L. Humphreys and J. B. Davies, "The synthesis of N-port circulators," IRE TRANS. ON MICROWAVE THEORY AND TECHNIQUES, vol. MTT-10, pp. 551–554; November, 1962.

Broad-Band Microwave Discriminator*

A sketch of a novel broad-band microwave discriminator is shown in Fig. 1. The device utilizes a pair of symmetric 3-db hybrids joined by unequal lengths of transmission line. The difference between line lengths is represented as a frequency dependent phase difference ϕ. A straightforward analysis of the circuit yields

$$|E_1| = 0.707E(1 + \cos\phi)^{1/2} \qquad (1)$$

$$|E_2| = 0.707E(1 - \cos\phi)^{1/2}. \qquad (2)$$

* Received April 8, 1963.